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Data compressibility, physical entropy, and evolutionary *a priori* relation between observer and object

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The thermodynamic and epistemological relevance of an evolutionary *a priori* relation between observer and object is analyzed in reference to Occam's razor and Zurek's physical entropy. It is demonstrated that the observer's *a priori* relation to the object determines the minimal demand for observed data. Therefore, physical entropy is relative and the maximal amount of net work that can be extracted depends on the observer. A formal quantitative analysis is presented using the concepts of algorithmic information theory. Zurek's algorithmic information distance is applied as an *in-principle* measure of the evolutionary *a priori* relation.

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INTRODUCTION

Considering the relation of observer and object has been of great heuristic value in physics. In a formal and fundamental way, the present article explores the thermodynamic and epistemological relevance of an evolutionary *a priori* relation. This relation may be due to common ancestors and/or interactions (of ancestors) in a common past; however, the concrete type of relation is not essential to the analysis performed here.

PHYSICAL ENTROPY AND DATA COMPRESSION

Recently, Zurek [1] defined the notion of physical entropy S_d as the sum of the missing information and of the algorithmic information content (= algorithmic complexity) of the available data: $S_d = H_d + K(d)$. This measure of entropy allows the fundamental limits of the amount of net work that an observer can extract from an observed system to be calculated. According to Zurek, the observer is a "complex adaptive system" operating as an "information gathering and using system" with the measuring and computing power of a Maxwell demon linked to a Turing machine.

The use of algorithmic information content in the definition of physical entropy has been induced (i) by Bennett's insight [2] that a Maxwell demon may indeed perform measurements and computation without dissipating energy, but that, nonetheless, it is limited by the second law of thermodynamics since it has to pay Landauer erasure costs when returning its memory to the premeasurement state, and (ii) by the demons in-principle ability to compress reversibly its postmeasurement description d of the observed system to the most compact version d^* [the length of which equals the algorithmic information content of d: $K(d) = |d^*|$] [1,3].

Zurek has shown that a decrease of physical entropy and an extraction of net work can be achieved only if the observed system contains a regular, that is, nonrandom configuration. Otherwise, the decrease of missing information after the demon's measurements is equalized by the algorithmic complexity of the resulting description in the demon's memory: $\Delta H = -\Delta K$. Observing regularities the demon can compress its description of the system and net work can be extracted.

DATA COMPRESSIBILITY AND *a priori* RELATION BETWEEN OBSERVER AND OBJECT

Additional compression of the observed data is possible if the demon can refer to *a priori* knowledge about the system under observation. Such *a priori* knowledge interferes in the case of observers and objects that are related evolutionarily. Without loss of generality, this will be outlined using three basic assumptions. (1) The system under observation contains a biological configuration C, which may be described by a string c having an algorithmic complexity $K(c)$. (Biological configurations show regularities but also have considerable complexity: They are neither simple like a crystal nor random like a gas [3].) (2) Noncomplex metaphysical demons do not exist. Zurek's demon [1] is a "complex adaptive system" with the computing power of a Turing machine. Biological systems are complex and adaptive. Furthermore, Turing machines can be simulated by biological configurations [2]. Hence, Zurek's demon can be implemented by a biological configuration. Such a biological demon M may be represented by a string m (premeasurement state). For reasons of simplicity, m may be assumed to be a minimal description, that is, $|m| = K(m)$. Biological computation dissipates energy; however, for the present purpose of in-principle analysis, M may be thought of operating reversibly. (3) Biological configurations are products of the evolution. The evolutionary relation of C and M implies that they share information, which may be expressed quantitatively as mutual algorithmic information content $K(c:m)$. According to Chaitin [3], $K(c:m)$ relates to the joint algorithmic complexity $K(c,m)$ of a concatenation of c and m as

$$K(c:m) = K(c) + K(m) - K(c,m) \ . \qquad (1)$$

After having performed an exhaustive measurement, M

contains the complete description c in its memory. Therefore, the postmeasurement state of M is described by the string (c,m). Zurek [1] has shown that in order to return to its premeasurement state, M has to pay at least erasure costs proportional to the conditional algorithmic information content $K((c,m)|m)$ of (c,m) given m; this conditional complexity represents the minimal amount of observed data that M needs in order to produce a complete description of C. [Notice that $K((c,m)|m)$ equals $K(c|m)$ since m is available.] Assuming m to be minimal, correction terms of order $O(\log K(m))$ vanish, and $K((c,m)|m)$ may be expressed as [1,3]

$$K((c,m)|m)=K(c,m,m)-K(m)=K(c,m)-K(m) \quad (2)$$

or, using Eq. (1), as

$$K((c,m)|m)=K(c)-K(c:m) \; . \quad (3)$$

Hence, as compared to a metaphysical demon or to any unrelated observer $[K(c:m_{\text{unrelated}}=0)]$ a biological demon can compress the observed data further by $K(c:m)$ units of information. If compression of observed data is understood as an application of Occam's razor, it follows that different biological observers cannot use the same razor since their relations to the observed object are different.

ALGORITHMIC INFORMATION DISTANCE BETWEEN OBSERVER AND OBJECT

Adding Eqs. (2) and (3) results in

$$K((c,m)|m)=[K(c,m)-K(c:m)+K(c)-K(m)]/2 \; . \quad (4)$$

The difference $K(c,m)-K(c:m)$ approximately [1] equals the information distance $\Delta(c,m)$ between c and m, which has been defined by Zurek:

$$K(c,m)-K(c:m)\approx\Delta(c,m)=K(c|m)+K(m|c) \; . \quad (5)$$

Equation (5) would be exact if c and m were both assumed to be minimal [1]. Using $\Delta(c,m)$, Eq. (4) may be expressed as

$$K((c,m)|m)\approx[\Delta(c,m)+K(c)-K(m)]/2 \; . \quad (6)$$

$\Delta(c,m)$ can be used as an in-principle measure of evolutionary distance between observer and object. According to Eq. (6), this distance determines the minimal amount of data that the observer M must acquire in order to describe an object C; or, in other words, this distance determines the maximal possible compression of data in the observers memory after its measurements.

EXTRACTION OF NET WORK AND *a priori* RELATION BETWEEN OBSERVER AND OBJECT

Due to the additional compression of the observed data by $K(c:m)$ units of information, an evolutionarily related demon can extract more net work from an object than unrelated or metaphysical demons since a large decrease of physical entropy is possible in case of such an *a priori* relation. However, by extraction of the maximal amount of energy, this *a priori* relation would be destroyed. Furthermore, the additionally extracted net work would be used up if the demon had to erase its *a priori* knowledge about the object. Therefore, the second law of thermodynamics is not endangered.

One might object that it is not possible to extract such additional net work since the *a priori* relation between observer and object also would decrease the initial amount of missing information H. However, before having performed its measurements the observer does not know whether and, if so, to what extent it has an *a priori* relation to the object under investigation. Only afterwards it can recognize this relation. Therefore, the additional compressibility of the observed data is not balanced by an antecedent decrease of missing information.

LIMITATIONS AND FUTURE INVESTIGATIONS

(1) The analysis above has been performed in terms of classical physics; especially the assumption of a complete description of an object has simplified the calculation. It may be asked whether a quantum mechanical approach (or, vice versa, an application to quantum mechanics) is reasonable. As long as the relation between observer and object is understood as a relation established by biological evolution, such an approach probably would not be of unexpected benefit. However, considering recent developments in quantum coding and theory of measurement may be interesting in case of other forms of observer-object relation.

(2) A concrete example could support the theoretical content of this report. However, observers and objects of high complexity have been considered here to an extent that correction terms of order $O(\log K(m))$ have been neglected. Therefore, simple detailed examples as found in other discussions of Maxwell demons cannot be applied. Modeling observer and object as assemblies of strings similar to nucleic acids may lead to a semirealistic biological example of adequate complexity. However, the search for biological examples is impaired by the fact that biological systems do not operate reversibly and by the large amount of entropy that is produced in biological energy metabolism. Therefore, while energy metabolism certainly requires some *a priori* knowledge and some measurements of the object, it will be difficult to show where biological systems touch upon the relativity of physical entropy reported in this paper.

CONCLUSION

Evolutionary *a priori* relations are of epistemological as well as thermodynamic relevance: The algorithmic information distance $\Delta(c,m)$ between observer M and object C can be regarded as an in-principle measure of their evolutionary distance and determines the minimal amount of data that the observer has to acquire in order to describe the object. As compared to an unrelated observer an *a priori* related observer can compress the observed data by extra $K(c:m)$ units of information, and consequently, due to the decrease of physical entropy, extract more net work from the observed object. However, this relativity of physical entropy usually is not detectable due to the large amount of entropy that is produced by biological systems.

[1] W. H. Zurek, Nature (London) **341**, 119 (1989); Phys. Rev. A **40**, 4731 (1989); in *Complexity, Entropy, and the Physics of Information,* edited by W. H. Zurek (Addison-Wesley, New York, 1990), pp. 73–89.

[2] C. H. Bennett, Int. J. Theor. Phys. **21**, 905 (1982).

[3] G. J. Chaitin, J. Assoc. Comput. Mach. **22**, 329 (1975); in *The Maximum Entropy Formalism,* edited by R. D. Levine and M. Tribus (MIT, Cambridge, MA, 1979), pp. 477–498.